Chapter 5
A Special Wohlhart's Double-Goldberg 6*R* Linkage and its Multiple Operation Forms Among 4*R* and 6*R* Linkages

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Abstract For Wohlhart's double-Goldberg 6*R* linkage constructed from two original Goldberg 5*R* linkages, there are two constructive forms and one non-constructive form, which can be reconfigured through bifurcation points. A special Wohlhart's double-Goldberg 6*R* linkage is proposed by introducing a special geometry constraint. One of the constructive forms of the resultant linkage is degenerated into a pseudo 4*R* linkage, which is essentially a Bennett linkage. Therefore, the special Wohlhart's double-Goldberg 6*R* linkage achieves multiple forms among 4*R* and 6*R* linkages. Physical models are also made to validate the special Wohlhart's double-Goldberg 6*R* linkage in different operation forms.

Keywords Wohlhart's double-Goldberg linkage · Overconstrained linkage · Multiple operation modes

5.1 Introduction

The concept and design of reconfigurable mechanisms have been developed to meet the need of the emerging new frontiers of multi-functional machines and robots since 1990s. One effective method of reconfiguration is through the bifurcation points on kinematic paths of different linkages. The kinematotropic linkage proposed by Wohlhart [1] can even change its global mobility with positional parameter actuations at the bifurcation points. Galletti and Fanghella [2]

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used the displacement groups method and designed a series of kinematotropy mechanisms. The metamorphic mechanisms were proposed by Dai and Rees [3] with variable topology and mobility. Recently, Kong and Huang [4] proposed a number of one degree-of-freedom single-loop overconstrained linkages with two operation modes by using type synthesis method. Wohlhart [5] also proposed a series of multifunctional 7*R* linkages by inserting an overconstrained 4*R*, 5*R* or 6*R* mobile chain into a closed-loop 7*R* linkage. In this paper, a special geometry condition is introduced to Wohlhart's double-Goldberg 6*R* linkage to make it have multiple operation forms among 4*R* and 6*R* linkages.

This paper is organized as follows. Sect. 5.2 introduces three forms of Wohlhart's double-Goldberg 6*R* linkage. In Sect. 5.3, the original form of Wohlhart's double-Goldberg 6*R* linkage is degenerated into a Bennett linkage. Section 5.4 describes the multiple operation forms and their transitions. The conclusion is enclosed in the last section. In this paper, $a_{(i-1)i}$ is length of link $(i-1)i$, $\alpha_{(i-1)i}$ is twist of link $(i-1)i$, R_i is offset of joint i and θ_i is revolute variable of joint i. a/α, b/β, c/γ and d/δ are the lengths and twists of the link, e.g., a/α is a link with length a and twist α.

5.2 Wohlhart's Double-Goldberg 6*R* Linkage

Recently, Wohlhart's double-Goldberg 6*R* linkage [6] was re-examined by Song and Chen [7]. Shown in Fig. 5.1 are the two constructive forms of Wohlhart's double-Goldberg 6*R* linkage. Both linkages are obtained by merging two Goldberg 5*R* linkages on the commonly shared link-pair $a/\alpha \sim c/\gamma$. After removing the common links and joint in dash lines, the rest part will form the Wohlhart's double-Goldberg 6*R* linkage. The one in Fig. 5.1a was originally formed by Wohlhart and named as Form I linkage. The one in Fig. 5.1b is the newly found constructive form [7] and named as Form II linkage. During bifurcation analysis, the third linkage form was detected and named as Form III linkage. The Form III linkage cannot be decomposed into the combination between two Goldberg 5*R* linkages as Forms I and II, and therefore it is non-constructive. All three forms of Wohlhart's double-Goldberg 6*R* linkage share the identical geometry conditions as Eqs. (5.1a), (5.1b), (5.1c) and (5.1d) yet have different closure equations.

$$a_{12} = a_{45} = a + c,\ a_{23} = a_{61} = b,\ a_{34} = a_{56} = d, \tag{5.1a}$$

$$\alpha_{12} = \alpha_{45} = \alpha + \gamma,\ \alpha_{23} = \alpha_{61} = \beta,\ \alpha_{34} = \alpha_{56} = \delta, \tag{5.1b}$$

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c} = \frac{\sin\delta}{d}, \tag{5.1c}$$

$$R_i = 0\ (i = 1,\ 2,\ \ldots,\ 6). \tag{5.1d}$$

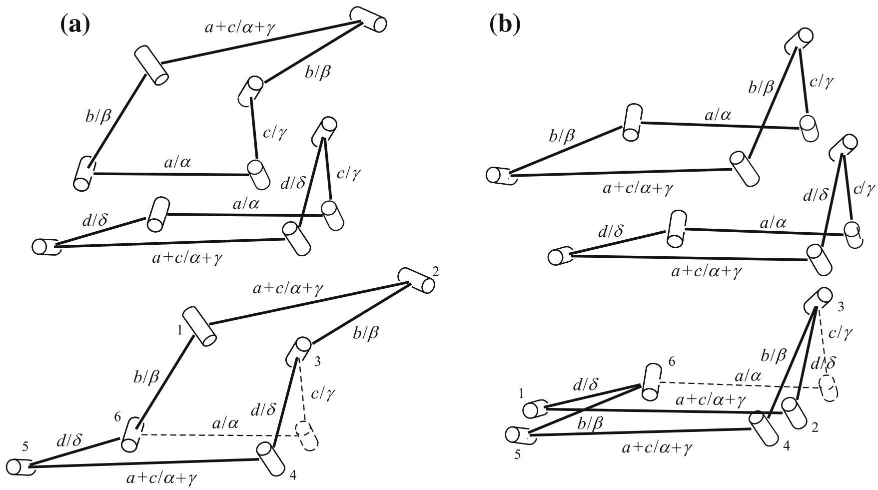


Fig. 5.1 Two constructive forms of Wohlhart's double-Goldberg 6*R* linkage: **a** original Form I linkage reported by Wohlhart [6] and **b** new constructive Form II linkage reported by Song and Chen [7]

The closure equations of Forms I and II linkages can be derived from their construction process [7], which are

$$\begin{gathered}\tan\tfrac{\theta_2}{2}=\frac{m_1m_2}{\tan\frac{\theta_1}{2}},\ \theta_3=2\tan^{-1}\left(\frac{m_1}{\tan\frac{\theta_1}{2}}\right)+2\tan^{-1}P_{\theta_1}-\pi\\ \tan\frac{\theta_4}{2}=m_4P_{\theta_1},\ \tan\frac{\theta_5}{2}=\frac{m_3}{P_{\theta_1}},\ \theta_6=-\theta_3,\end{gathered}, \tag{5.2}$$

in which

$$m_1=\frac{\sin\dfrac{\beta+\alpha}{2}}{\sin\dfrac{\beta-\alpha}{2}},\ m_2=\frac{\sin\dfrac{\beta+\gamma}{2}}{\sin\dfrac{\beta-\gamma}{2}},\ m_3=\frac{\sin\dfrac{\delta+\alpha}{2}}{\sin\dfrac{\delta-\alpha}{2}},\ m_4=\frac{\sin\dfrac{\delta+\gamma}{2}}{\sin\dfrac{\delta-\gamma}{2}} \tag{5.3}$$

For Form I linkage

$$\begin{gathered}q=|m_1|\sqrt{\frac{(m_2+m_3)-m_2m_3(m_1+m_4)}{m_1m_4(m_2+m_3)-(m_1+m_4)}},\ Q=\frac{(m_1m_2-1)\tan\frac{\theta_1}{2}}{m_1m_2+\tan^2\frac{\theta_1}{2}}\\ P_{\theta_1}=\begin{cases}\dfrac{(1-m_3m_4)+\sqrt{(1-m_3m_4)^2-4m_3m_4Q^2}}{2m_4Q} & \left(\begin{matrix}-\pi\le\theta_1\le-2\tan^{-1}q\\ 2\tan^{-1}q\le\theta_1\le\pi\end{matrix}\right)\\ \dfrac{(1-m_3m_4)-\sqrt{(1-m_3m_4)^2-4m_3m_4Q^2}}{2m_4Q} & (-2\tan^{-1}q\le\theta_1\le 2\tan^{-1}q)\end{cases},\end{gathered} \tag{5.4}$$

and for Form II linkage, change P_{θ_1} into the following,

$$P_{\theta_1} = \begin{cases} \dfrac{(1 - m_3 m_4) + \sqrt{(1 - m_3 m_4)^2 - 4 m_3 m_4 Q^2}}{2 m_4 Q} & (-2\tan^{-1} q \le \theta_1 \le 2\tan^{-1} q) \\ \dfrac{(1 - m_3 m_4) - \sqrt{(1 - m_3 m_4)^2 - 4 m_3 m_4 Q^2}}{2 m_4 Q} & \begin{pmatrix} -\pi \le \theta_1 \le -2\tan^{-1} q \\ 2\tan^{-1} q \le \theta_1 \le \pi \end{pmatrix} \end{cases} \tag{5.5}$$

As the Form III linkage is non-constructive, its closure equations cannot be derived using the same method as Forms I and II. However, we can use the singular value decomposition method [8] to obtain its kinematic paths numerically.

5.3 A Special Wohlhart's Double-Goldberg 6*R* Linkage

From Eqs. (5.2), (5.4) and (5.5), θ_3 and θ_6 are not equal to zero generally during the motion. If constraining $\theta_3 = \theta_6 = 0$, we will get

$$\frac{(1 - m_3 m_4) \pm \sqrt{(1 - m_3 m_4)^2 - 4 m_3 m_4 \left[\dfrac{(m_1 m_2 - 1)\tan\frac{\theta_1}{2}}{m_1 m_2 + \tan^2\frac{\theta_1}{2}}\right]^2}}{2 m_4 \dfrac{(m_1 m_2 - 1)\tan\frac{\theta_1}{2}}{m_1 m_2 + \tan^2\frac{\theta_1}{2}}} = \frac{\tan\frac{\theta_1}{2}}{m_1}, \tag{5.6}$$

i.e.,

$$[m_4(m_1 m_2 - 1) - m_1(1 - m_3 m_4)]\tan^2\frac{\theta_1}{2} = m_1^2[m_2(1 - m_3 m_4) - m_3(m_1 m_2 - 1)] \tag{5.7}$$

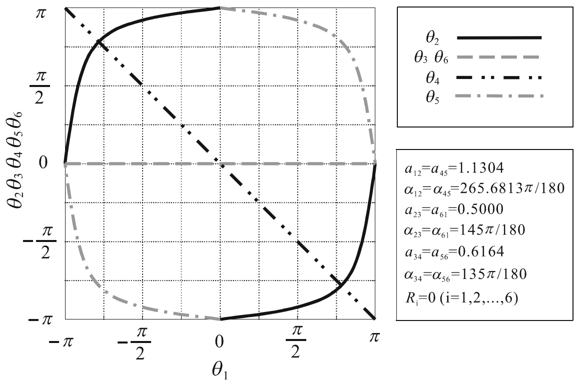
Here, $m_i (i = 1,\ 2,\ 3,\ 4)$ in Eq. (5.3) are fixed for a given linkage. Thus, in order to make θ_1 free, Eq. (5.7) can be held only when

$$\begin{cases} m_4(m_1 m_2 - 1) - m_1(1 - m_3 m_4) = 0 \\ m_2(1 - m_3 m_4) - m_3(m_1 m_2 - 1) = 0 \end{cases} \tag{5.8}$$

One solution of Eq. (5.8) is

$$\begin{cases} m_4 = -m_1 \\ m_3 = -m_2 \end{cases}, \tag{5.9}$$

Fig. 5.2 Kinematic paths of the special Wohlhart's double-Goldberg 6*R* linkage in Bennett form



which is

$$\tan\frac{\alpha}{2}\tan\frac{\gamma}{2} = \tan\frac{\beta}{2}\tan\frac{\delta}{2}, \tag{5.10}$$

in terms of link twists. Furthermore, the following relationship can be obtained by considering Eqs. (5.1c) and (5.10) that

$$\frac{\sin(\alpha+\gamma)}{a+c} = \frac{\sin(\beta+\delta)}{b+d}. \tag{5.11}$$

Therefore, when the condition shown in Eq. (5.11) is introduced to Eq. (5.1), a special Wohlhart's double-Goldberg 6*R* linkage will be formed, whose revolute variables θ_3 and θ_6 are expected to be zero during full-circle motion. In other words, θ_3 and θ_6 are kinematically constrained to zero during the full-circle motion. The closure equations of Forms I and II of the special Wohlhart's double-Goldberg 6*R* linkage can be obtained as below by substituting Eq. (5.11) into Eqs. (5.2), (5.4) and (5.5) that

$$\tan\frac{\theta_2}{2} = \frac{m_1 m_2}{\tan\frac{\theta_1}{2}},\ \tan\frac{\theta_4}{2} = -m_1 P_{\theta_1},\ \tan\frac{\theta_5}{2} = -\frac{m_2}{P_{\theta_1}}, \tag{5.12a}$$

$$\theta_3 = 2\tan^{-1}\left(\frac{m_1}{\tan\frac{\theta_1}{2}}\right) + 2\tan^{-1}P_{\theta_1} - \pi,\ \theta_6 = -\theta_3, \tag{5.12b}$$

in which

$$P_{\theta_1} = \begin{cases} \dfrac{\tan\frac{\theta_1}{2}}{m_1} & \text{for Form I linkage,} \\ \dfrac{m_2}{\tan\frac{\theta_1}{2}} & \text{for Form II linkage.} \end{cases} \tag{5.13}$$

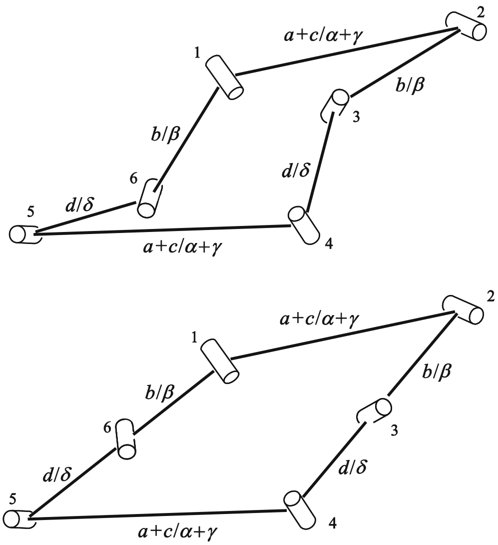


Fig. 5.3 Reconfiguration of Form I of Wohlhart's double-Goldberg 6*R* linkage into a Bennett 4*R* linkage

Only for Form I linkage, $\theta_3 = \theta_6 = 0$ after introducing the constraint in Eq. (5.11) to the 6*R* linkage. Thus, this 6*R* linkage is degenerated into a pseudo 4*R* linkage, whose kinematic paths are shown in Fig. 5.2.

After careful examining the geometry conditions of the special Wohlhart's double-Goldberg 6*R* linkage and its corresponding kinematic closure equations with $\theta_3 = \theta_6 = 0$, it is interesting to find that the pseudo 4*R* linkage is essentially a Bennett linkage connected by joints 1, 2, 4 and 5 with links b/β and d/δ adjacent to joints 3 and 6 rigidified into a composite link $b + d/\beta + \delta$, see Fig. 5.3.

5.4 Multiple Operation Forms of the Special Wohlhart's Double-Goldberg 6*R* Linkage

Generally, Wohlhart's double-Goldberg 6*R* linkage has three forms: two constructive 6*R* forms and one non-constructive 6*R* form with different kinematic paths, which can transform into each other through bifurcation points [7]. After introducing the special geometry constraint in Eq. (5.11), the Form I linkage, degenerates to a Bennett form with only four active revolute variables. Thus, the operation form of a 4*R* linkage has been successfully introduced to Wohlhart's double-Goldberg 6*R* linkage.

The transition among different forms of the special Wohlhart's double-Goldberg 6*R* linkage is shown in Fig. 5.4. Only the relationship between θ_1 and θ_5 is used for the ease of representation. The black solid lines correspond to the Form II of Wohlhart's double-Goldberg linkage in constructive 6*R* form. The grey solid lines correspond to the Form I of Wohlhart's linkage in Bennett 4*R* form.

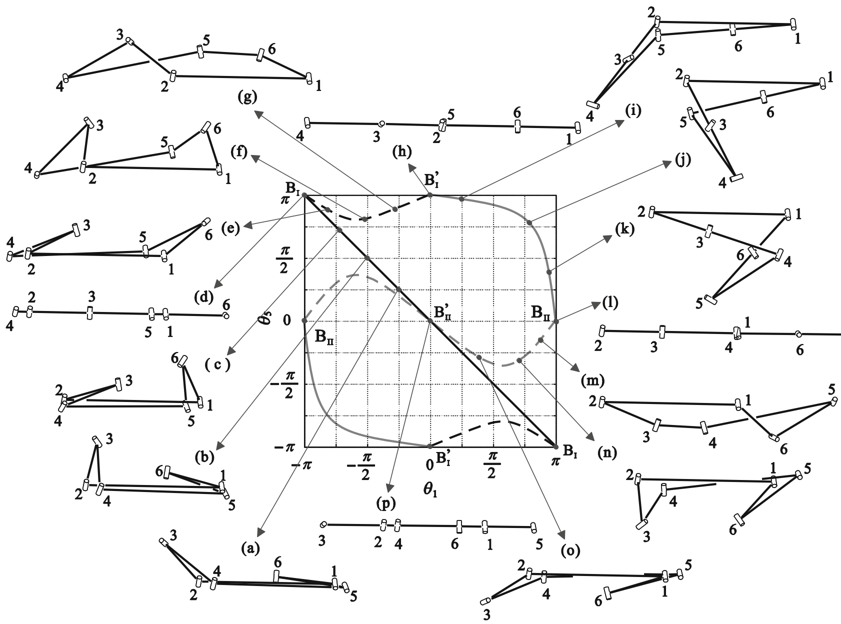


Fig. 5.4 Transitions of the special Wohlhart's double-Goldberg 6*R* linkage in different operation forms: (*a*) ~ (*c*) are in constructive 6*R* form; (*e*) ~ (*g*) and (*m*) ~ (*o*) are in different configurations of non-constructive 6*R* form; (*i*) ~ (*k*) are in Bennett 4*R* form. (*d*), (*h*), (*l*) and (*p*) are the transition configuration at bifurcation points

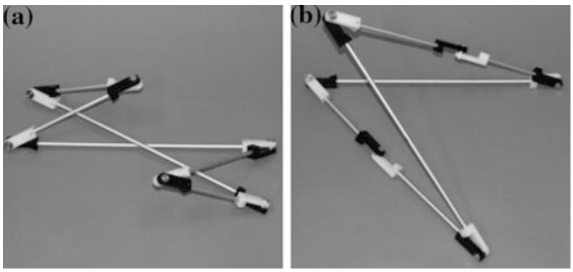


Fig. 5.5 Physical models of the special Wohlhart's double-Goldberg 6*R* linkage. **a** Constuctive 6*R* form **b** Bennett 4*R* form

The black and grey dash lines represent the Form III of Wohlhart's double-Goldberg linkage in different configurations, which are in non-constructive 6*R* forms [7]. B'_{I}, B_{II} and B'_{II} are the bifurcation points between different forms. Physical models of the special Wohlhart's double-Goldberg 6*R* linkage are also presented in Fig. 5.5.

5.5 Conclusion

In this paper, a special Wohlhart's double-Goldberg 6*R* linkage is presented to have multiple operation forms among 4*R* and 6*R* linkages. Originally, the Wohlhart's double-Goldberg 6*R* linkage has three operation forms: Forms I and II are in constructive 6*R* forms; and Form III is in non-constructive 6*R* form. By investigating the closure equations, a special geometry condition that $\sin(\alpha+\gamma)/(a+c) = \sin(\beta+\delta)/(b+d)$ is introduced, resulting two revolute variables to be kinematically inactive in Form I linkage. The Form I linkage degenerates into a Bennett linkage. Therefore, the special Wohlhart's double-Goldberg 6*R* linkage has three operation forms: a constructive 6*R* form, a non-constructive 6*R* form and a Bennett 4*R* form. Different forms of the linkage can be transited into each other through bifurcation points.

The work in this paper demonstrates the possibility and approach to propose reconfigurable mechanisms among kinematic loops with different number of links. Such mechanisms will play a fundamental role in the multifunctional robots and manufacture system of new generation.

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